



TITLE:

McKay quiver representations and tiltings

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RIGHT:

McKay quiver representations and tiltings

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Aim

(1) Describe : $\mathcal{M}_{C,d}(Q, R) \xrightarrow{\sim} \mathcal{M}_{C',d}(Q, R)$

tilting theory.

The isomorphism of moduli spaces.

(2) Generalize : **Crawley-Boevey's theorem.**

Moduli spaces and Representations

Notations

G : a finite subgroup of $SL(2, \mathbb{C})$.

$\{\rho_0, \dots, \rho_n\}$: the set of irreducible reps of G .

(Q, R) : the McKay quiver of G .

$\mathbf{d} := (\dim \rho_i)_{i \in Q_0}$: a dimension vector
 C : a chamber in the stability space $\left. \vphantom{\begin{matrix} \mathbf{d} \\ C \end{matrix}} \right\} \xrightarrow{GIT} \mathcal{M}_{C,d}(Q, R)$.

$\text{rep}_{C,d}(Q, R)$: the category of \mathbf{d} -semistable (Q, R) reps
w.r.t. $\in C$ of dimension vector \mathbf{d} .

$(\text{rep}_{C,d}(Q, R) / \simeq) = \mathcal{M}_{C,d}(Q, R)$ as a set.

Fact

(Kronheimer)

$\mathcal{M}_{C,d}(Q, R)$ is the minimal resolution.

(Nakajima, Lusztig etc.)

Describe : $S : \mathcal{M}_{C,d}(Q, R) \xrightarrow{\sim} \mathcal{M}_{C',d}(Q, R)$ as a reflection functor.

Tilting theory and Weyl groups

Tilting modules

Λ : a noetherian algebra (e.g. $\mathbb{C}Q/R$).

T : a right Λ -module.

T : tilting $\Leftrightarrow \begin{cases} (1) \text{proj.dim}_{\Lambda} T \leq 1 \\ (2) \text{Ext}_{\Lambda}^1(T, T) = 0 \\ (3) \exists \text{ exact sequence} \\ 0 \rightarrow \Lambda \rightarrow T_1 \rightarrow T_2 \rightarrow 0 \quad (T_i \in \text{add } T) \end{cases}$

Theorem(Happel, Rickard)

Tilting modules induce the equivalences

$$\mathcal{D}^b(\text{mod } \Lambda) \simeq \mathcal{D}^b(\text{mod } \text{End}_{\Lambda}(T)).$$

Theorem (Iyama-Reiten)

$\mathbb{C}Q/R$: the preprojective algebra.

W : the corresponding Weyl group.

$I_i := \langle 1 - e_i \rangle$: an ideal of $\mathbb{C}Q/R$.

$\mathcal{I} := \{I_{i_1} \cdots I_{i_k} \mid 1 \leq i_1, \dots, i_k \leq n, k \in \mathbb{N}\}$: a set of ideals of $\mathbb{C}Q/R$.

(1) $\mathcal{I} \ni \forall T$ is a tilting module over $\mathbb{C}Q/R$ s.t.
 $\text{End}_{\Lambda}(\mathbb{C}Q/R) \cong \mathbb{C}Q/R$.

(2) \exists a one-to-one correspondence $W \rightarrow \mathcal{I}$;

$$w = s_{i_1} \cdots s_{i_k} \mapsto I_w := I_{i_1} \cdots I_{i_k} \quad (\text{a reduced expression})$$

Main Theorem

C_G : fix the chamber defined by $\theta_i < 0$ and $\theta_i > 0$ ($i \neq 0$).
(i.e. $\mathcal{M}_{C_G,d} = G\text{-Hilb}$).

Main Theorem

There is a category equivalence

$$\text{Hom}_{\mathbb{C}Q/R}(I_w, -) : \text{rep}_{C_G,d}(Q, R) \xrightarrow{\sim} \text{rep}_{C_G^w,d}(Q, R)$$

which induces the isomorphism

$$\mathcal{M}_{C_G,d}(Q, R) \xrightarrow{\sim} \mathcal{M}_{C_G^w,d}(Q, R).$$

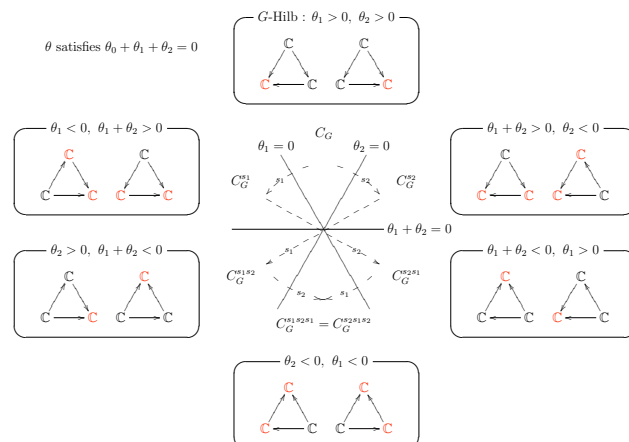
Point of proof of Main Theorem

$$\mathbb{R} \text{Hom}_{\mathbb{C}Q/R}(I_i, -) : \mathcal{D}^b(\text{mod } \mathbb{C}Q/R) \xrightarrow{\sim} \mathcal{D}^b(\text{mod } \mathbb{C}Q/R)$$

$$\cup \qquad \cup$$

$$\text{Hom}_{\mathbb{C}Q/R}(I_i, -) : \text{rep}_{C,d}(Q, R) \xrightarrow{\sim} \text{rep}_{C^{s_i},d}(Q, R)$$

Example of A_2 -type



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